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SOVIET DEVELOPMENT IN INFORMATION PROCESSING AND MACHINE TRANSLATION

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SOVIET DEVELOPMENTS IN INFORMATION PROCESSING AND MACHINE TRANSLATION

FOREWORD

This translation series presents information from Soviet literature on developments in the following fields in information processing and machine translation: organization, storage and retrieval of information; coding; programming; character and pattern recognition; logical design of information and translation machines; linguistic analysis with machine translation application; mathematical and applied linguistics; machine translation studies. The series is published as an aid to U. S. Government research.

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JPRS: 68, 241, 319, 355, 379, 387, 487, 621, 646, 662, 705, 729, 863, 893, 925, 991, 992, 1006, 1029, 1130, 1131, 1132, 1133, 3225, 3300, 3356, 3502, and 3532.

THE QUANTITY OF COORDINATE DESCRIPTIONS OF IMAGES IN SYSTEMS FOR RECOGNITION OF VISIBLE PATTERNS (CHARACTER RECOGNITION DEVICES)

/Following is a translation of an article by V. S. Fayn in the Russian-language periodical Izvestiya Akademii Nauk SSSR -- Otdeleniye tekhnicheskikh nauk (Bulletin of the Academy of Sciences USSR -- Department of Technical Sciences), No. 2, Moscow, 1960, pages 164-172./

The most important part of a machine for recognizing visible patterns is the viewing apparatus which transforms light signals into electrical or other disturbances. In those cases in which this apparatus is similar to the retina of the living eye, the perception of a continuous image is accomplished by its discretization and the number of discrete elements of decomposition is equal to the number of elements in the retina. Information on the degree of illumination and the coordinates of the elements of the retina on which the image is projected is sent to the "thinking" part of the machine for further realization of recognition. Recognition takes place as a result of synthesis of the lines from individual elements of decomposition and synthesis itself is realized on the basis of comparison of the coordinates of the elements of decomposition of the same or differing (in a determined manner) brightness. We shall call the combination of coordinates of the elements forming the contour of the image the coordinate description of the image. It is obvious that different coordinate descriptions of the same image will be received, depending on the size and the distribution of the projection of the image on the retina.

Recognition of a given image should not depend on which of its coordinate descriptions has been formed by the retina. Therefore it will be of interest to determine the quantity N coordinate descriptions of the same image and the interrelationship of N and the dimensions of the retina.

Let us introduce some definitions.

We shall call the distance between the two most widely separated points of its contour the largest dimension or over-all size of the image D (Figure 1).

Depending on the configuration formed by the contour, we shall call the smallest of the following quantities the smallest dimension of the image d:

1) The smallest radius of curvature (Figure 1a);

2) The smallest distance between two break points (Figure 1b);

3) The smallest distance between two different parts of the configuration -- in case an image formed by convexo-concave curves contains clear-cut individual parts (outgrowths, curved configurations) and the distance between these parts has a minimum not at the place of merger of the parts (Figure 1c).

Let us call the quantity $\alpha = D/d$ the coefficient of detailedness of the image or simply detailedness.

Let us assume that a set of elements of decomposition can be perceived as a line segment if this segment is projected on not less than a elements of the retina. Moreover, we shall consider the elements of the retina scattered evenly over its surface. In this case it will be convenient to measure the distance on the retina in quantities of elements of the retina packed into the given line. Here the minimum scale D_{jmin} at which it is still possible to resolve the smallest details of the image is obtained when d = a. Then the value of the over-all size, expressed in elements of the retina, is equal (for this minimum scale) to

$$D_{min} = \alpha a \text{ elements of the retina} \qquad (1)$$

It is important for what is to follow that if several images have an equal value of detailedness α, then when taken at any scale, but the same for all of them (including the minimum too), they have the same over-all size, that is, if $\alpha_1 = \alpha_2$

$$D_{1j} = D_{2j} \qquad (1')$$

for all possible j, beginning with j_{min}.

The size of retina A should be larger than the value of D_{min} in order to ensure the possibility of increasing the size of the image or shifting it about on the retina with leaving the bounds of the retina. It follows that

$$A = \alpha\beta a \qquad (2)$$

where β is the "coefficient of distribution" on the retina (set by the initial conditions).

Later we shall consider a retina of a square form with sides of A (elements) in order to simplify calculations.

Now it is possible to pass on to an examination of the problem. The problem of the quantity of N possible coordinate descriptions of the same image was examined in other works /1, 2/, but the authors did not take into consideration that not every line projected on the retina forms the same image.

Let us see how different coordinate descriptions are formed of the same image.

Isomorphic transformations can be performed in the plane of the retina which belong to the following three groups:

1) The group of translations;

2) The group of rotations about a single arbitrary point;

3) The group of changes in scale.

The number of all possible coordinate descriptions is the sum of the quantities of transformations accomplished by these groups, taking into account the fact that the scales of the transformations are discrete. The latter means that

1) In translation the image can occupy only discrete positions determined by a whole number of units of linear measure (elements of the retina);

2) In rotations the image can occupy only positions such that the point of the image most distant from the center of rotation is displaced by a whole number of linear units;

3) In changes of scale of the image, when one of two of the points most widely separated from each other on the contour is fixed, the second can be displaced by a whole number of linear units.

We note that from the standpoint of the quantity of possible coordinate descriptions the selection of the center of rotation is arbitrary since any position of the image achieved by rotating it about one center can be accomplished with rotation about any other center and the proper translation. However, for the sake of definiteness in further discussion, we shall always consider the center of rotation to be located at one of the ends of the line of the over-all size (or at one of the ends of any such lines if it is possible to draw several in the configuration) (Figure 1).

All the foregoing permits outlining a general way to determine the quantity of N coordinate descriptions of some image.

At first one determines the possible quantity of translations of the image with the condition it has the ith angle of rotation and a scale such that the over-all size has the value D_j (in view of the previously discreteness of transformations, all angles of rotation can be enumerated for every D_j just as all D_j can be enumerated for each angle of rotation).

It is easy to see that this quantity of translations is equal to

$$N_{ij} = (A - x_{ij})(A - y_{ij}) \tag{3}$$

where x_{ij} and y_{ij} are projections of the image on the x and y axes, respectively. The complete quantity of N coordinate descriptions can be found by double summing of N_{ij} over all i and j. In order to carry out such summing, it is essential to know the pattern of $x_{ij} = x(i,j)$ and $y_{ij} = y(i,j)$. The character of these patterns is wholly determined

by the shape of the image and, consequently, the quantity of coordinate descriptions N is determined by the form of the image, that is, $N = N[x(i,j), y(i,j)]$. In other words, with the same detailedness α_k images of different shapes will yield different quantities of coordinate descriptions.

It follows from this that no general expression exists which determines the quantity of coordinate descriptions irrespective of the shape of the image. On the other hand, however, this gives rise to the following Theorem 1 (the existence theorem).

> Among all the possible configurations possessing the same detailedness α, there exist at least two such configurations, to be called limit configurations henceforth, of which one yields the largest (upper limit configuration) and the second the smallest (lower limit configuration) quantity of coordinate descriptions.

The theorem formulated here stems logically out of the considerations preceding it. Its formal proof, which is quite simple and is not of interest from the standpoint of the problem under discussion, will not be given here.

One can draw the following conclusions on the basis of the foregoing discussion:

1) The problem examined in this paper of finding the quantity of N coordinate descriptions of any configuration of detailedness α can be solved only by stating the limits in which N is to be found. It is obvious that these limits should depend only on α (or on quantities which in turn are functions of only α, for example A for given a and β).

2) $N = N[x(i,j), y(i,j)]$ and the functions $x(i,j)$ and $y(i,j)$ are determined by the shape of the image. The problem of finding the limits is the problem of finding at least two pairs of functions $x(i,j)$ and $y(i,j)$ which yield the maximum and the minimum for the functional $N[x(i,j), y(i,j)]$, respectively, with determined additional conditions, that is, the variational problem.

In this case the solution proceeds along the line of setting requirements and the subsequent search for functions which satisfy these requirements.

A second procedure is also possible which is the inverse of the first. In this case some configurations are first selected with functions $x(i,j)$ and $y(i,j)$ corresponding to them, then their correspondence with the given requirements is verified. In case the selection of configurations is made to take into account any considerations which eliminate the necessity of a set containing all possible configurations of detailedness α the second procedure is preferable since the structure of the functional N, which contains a double summation of the expressions, including the three-dimensional functions $x(i,j)$ and $y(i,j)$

makes the application of a different type of mathematical operations, including the operations of the calculus of variations, quite difficult.

Let us examine the expression (3). The index j in this expression means that all configurations for which (3) is calculated are regarded as having the same scale, the jth scale, that is, they have the same over-all size D_j (on the strength of (1')).

It is obvious that $x_{ij} \leq D_j$, $y_{ij} \leq D_j$ is always true and that the larger x_{ij} and y_{ij} are, the smaller N_{ij} will be, that is, with a given angle of rotation i, the number of translations N_{ij} is less for that configuration for which x_{ij} and y_{ij} are closest to D_j. This leads to the assumption that the lower limit configuration may be a circle of diameter D_{ij} since $x_{ij} = y_{ij} = D_j$ hold for it for all i.

Analogous reasoning leads to the thought that the upper limit of configuration may be a segment of a straight line of length D_j. However, prior to undertaking the proof that the circle and the straight line segment are limit configurations, it is essential to discuss some of their geometrical properties.

1) The equality of the detailedness of the limit configuration of the quantity α is a condition of the application of some configurations as limit configurations for all arbitrary configurations of detailedness α. This ensures the possibility of changing the scale of the limit configuration within those and only those limits determined by the quantity in which the scale of the arbitrary configurations can be changed. In this case, for every over-all size of an arbitrary configuration of detailedness α it is possible to find an equal over-all size in the limit configuration, that is, the arbitrary and limit configurations have the same range of changes j.

We note now that the circle and the straight line segment have detailedness $\alpha' = 1$. In order to make it possible to apply these configurations as limit configurations without accompanying these configurations on every occasion of their application with instructions on the range of changes in scale, it is possible, for the sake of simplicity, artifically to assign to them the same value of detailedness α possessed by the arbitrary configurations of the given set.

2) When a circle is rotated about its geometric center with any angle of rotation, the same coordinate descriptions are obtained. When a straight line segment is rotated 180 degrees the same coordinate descriptions are obtained. In general, if a configuration has an n-ray central symmetry, then every coordinate description of the configuration is repeated n times in one 360-degree rotation about the center of symmetry.

At the same time, any configurations which do not possess central symmetry yield different coordinate descriptions for all angles of rotation when rotated about any center. Therefore, when applying the circle and the straight line segment as limit configurations it is necessary to remember for the sake of generality that these configurations do not possess central symmetry.

Let us summarize all the foregoing in the following manner. Let us examine the set of all possible configurations which have the same value of detailedness α_k. We shall introduce into this set a series of auxiliary configurations to which we shall artificially assign the same value of detailedness α_k irrespective of their real geometrical properties. We shall call these configurations special configurations. Thus, for special configurations (OK)

$$\alpha(OK) = \alpha_k \tag{4}$$

Moreover, for the sake of generality we shall assume that all configurations of the set, including the special configurations too, do not possess central symmetry.

Finally, we note that we can require recognition of an image only as long as it remains within the limits of the retina or touches its boundary at no more than one point. Thus, for a given ith angle of rotation the jth scale is the maximum, at which time

$$x_{ij} = A - 1 \text{ or } y_{ij} = A - 1 \tag{5}$$

We shall make the idea of the index for numbering the scale j more precise. The most simple and natural numbering is obtained when the number of the scale is equal to the over-all length expressed in elements of the retina, that is,

$$j = D_j \tag{6}$$

The following Theorem 2 can be formulated on these assumptions.

The quantity N of all possible coordinate descriptions of any configuration is included within the limits expressed by the following inequalities:

$$N(OO) \leqq N(IK) \leqq N(O\pi) \tag{7}$$

where $N(OO)$, $N(IK)$, and $N(O\pi)$ are the quantities of the coordinate descriptions of the special circle (OO), the configuration under study (IK), and the special straight line segment ($O\pi$), respectively.

In other words, the theorem states that the above special configurations are limit configurations.

The proofs of the left and right parts of expression (7) can be completed in turn. In particular, the correctness of the inequality $N(OO) \leqq N(IK)$ is proved in the appendix.

By making use of the expressions (16) and (25) (refer to the appendix), it is possible to rewrite inequality (7), expressing the sense of the theorem in the following form:

$$\sum_{j=j_{min}}^{A-1} 2\pi D_j \ (A - D_j)^2 \leqslant N(NK \leqslant$$

$$\sum_{j=j_{min}}^{A-1} \sum_{i=0}^{2\pi D_j} (A - D_j \cos \frac{1}{D_j} i)(A - D_j \sin \frac{1}{D_j} i) \qquad (8)$$

Here D_j is written everywhere in place of $D_j(00)$ and $D_j(IK)$, since the limits of summation on j are the same in (16) and (25).

The quantity j_{min} contained in expression (8) is determined with the aid of expressions (6) and (1) through the detailedness $\propto(IK)$ of the configuration under study

$$j_{min} = D_{jmin} = \propto(IK)a$$

Consequently, knowing the size of retina A and the detailedness of the suggested configuration $\propto(IK)$, it is always possible to use expression (8) to calculate the limits in which the quantity of co-ordinate descriptions of this configuration is included, no matter what its form may be.

The quantity A may be quite large, thus, for definition corresponding to a television image, A is of an order of several hundreds. Here $f_{min} \ll A$ is usually the case.

In this case it is necessary to add up several hundreds of summands when summing on j, to say nothing of summing on i. For this reason the practical use of formula (8) is made difficult. In order to eliminate these difficulties it is expedient to make an approximate substitution of summation by integrating with the aid of the well known "quadrature formula"

$$\sum_{x=a}^{b} y_x \cong \frac{1}{\triangle} \int_a^b y dx \qquad (8a)$$

Here $\triangle$ is the base of an element of the triangle equal to the difference between values of x for two successive terms of the sum. In our case the x corresponds either to i, or to j and it is obvious that $\triangle = 1$ in both cases. Then

$$\sum_{x=a}^{b} y_x = \int_a^b y dx \qquad (9)$$

By applying (9) to (8), substituting D_j for j in accordance with (6), and assuming that $A \gg 1$, one can write

$$2\pi \int_{j_{min}}^{A} j(A - j)^2 dj \quad N(IK) \leq$$

$$\int_{j_{min}}^{A} dj \int_{0}^{2\pi j} (A - j \cos \frac{1}{j} i)(A - j \sin \frac{1}{j} i) di \tag{9a}$$

Carrying out the necessary calculations, we obtain

$$\pi[A^2(A^2 - j_{min}^2) - \frac{4}{3} A(A^3 - j_{min}^3) + \frac{1}{2} (A^4 - j_{min}^4)] \leq$$

$$N(IK) \leq \pi A^2(A^2 - j_{min}^2) \tag{10}$$

As already stated, usually $j_{min} \ll A$; this is all the more true of

$$j_{min}^2 \ll A^2, \; j_{min}^3 \ll A^3, \; j_{min}^4 \ll A^4$$

In evaluating this, the expression (10) can be simplified to the ultimate and can be written in the following form: $1/6\pi A^4 \leq N(IK) \leq A^4$ or by the integer $S = A^2$ elements of the retina

$$\frac{1}{6}\pi S^2 \leq N(IK) \leq \pi S^2 \tag{10'}$$

The quantity $1/6\pi S^2$ is the quantity of coordinate descriptions of the special circle (OO) which, as was proved, is the limit circle (π O). Likewise, the quantity πS^2 is the quantity of coordinate descriptions of the limit straight line segment ($\pi\pi$).

Thus, the quantities of coordinate descriptions of any, most varied configurations are distinguished among themselves by not more than 6 times. [One quantity cannot be more than 6 times any other such quantity.] This is due to the fact that on the jth scale the average projection of a straight line segment for all angles of rotation has a length of approximately 0.638 D_j, which is but little different from the length of a projection of a circle equal for all angles of rotation of D_j. Therefore the values of N_{ij} (refer to (3)) differ negligibly for the circle and the straight line segment.

If the configuration under study has an elongated shape, the quantity of its coordinate descriptions will approach $N(\pi\pi) = \pi S^2$. If, on the other hand, the configuration under study has dimensions nearly

the same in all directions, the quantity of its coordinate descriptions will approach $N(\Pi O) = 1/6\,\pi S^2$.

As an example, let us examine the quantity of coordinate descriptions of some image on a retina consisting of approximately 0.5×10^6 elements (the definition of a television image). In this case, it is obvious that

$$0.132 \times 10^{12} \leqq n(IK) \leqq 0.782 \times 10^{12}$$

It is interesting to note that a circle can be the lower limit configuration due to its property of having a constant value for the projection on any axis at all angles of rotation. However, the same property is possessed by any of the right curvilinear $(2n + 1)$-gons whose sides are formed by arcs of a circle, as shown in Figure 2a. All these figures are characterized by the fact that when one parallel plane is rolled over another, with the aid of such a figure (Figure 2b), the distance between these planes remains unchanged.

The circle which is the limit of such figures when $n \to \infty$ ensures the greatest simplicity of calculations. However, it is not difficult to show that the quantity of coordinate descriptions of any such figure (special) is equal to the number of coordinate descriptions of the special circle.

Appendix

Let us examine expression (7). Here $N(IK)$ is the quantity of coordinate descriptions of some arbitrary configuration under study (IK) of detailedness α_k; $N(00)$ is the quantity of coordinate descriptions of the special circle which has, in accordance with (4) the same detailedness $\alpha(00) = \alpha_k$.

In accordance with (3) we have for the ith angle of rotation and the jth scale

$$N_{ij}(00) = [A - x_{ij}(00)][A - y_{ij}(00)]$$
$$N_{ij}(IK) = [A - x_{ij}(IK)][A - y_{ij}(IK)] \qquad (10a)$$

But, the projections of a circle on any axis lying on its plane are equal to its diameter $D_j(00)$ and consequently the following holds true for all i:

$$x_{ij}(00) = y_{ij}(00) = D_j(00)$$

and $N_{ij}(00) = [A - D_j(00)]^2$ irrespective of the number i of the angle of rotation.

For the IK configuration which has been studied, the quantity $D_j(IK)$ is by definition the largest dimension; consequently, just two

such angles of rotation φ_i' and φ_i' + 180 degrees are found for which the following hold true

$$\begin{aligned} x_{ij}(IK) &< D_j(IK) \\ y_{ij}(IK) &\leqslant D_j(IK) \end{aligned} \tag{11}$$

and two angles φ_i' + 90 degrees and φ_i' + 270 degrees for which the following hold

$$\begin{aligned} y_{ij}(IK) &< D_j(IK) \\ x_{ij}(IK) &\leqslant D_j(IK) \end{aligned} \tag{11'}$$

However, in carrying out (4), (1) holds true and consequently for any arbitrary jth scale we have

$$D_j(00) = D_j(IK) \tag{12}$$

where $D_j(00)$ and $D_j(IK)$ are the over-all sizes of the special circle and the configuration under study, respectively.

Then, for the above angles it is obvious that we have

$$\begin{aligned} N_{ij}(IK) = [A - x_{ij}(IK)][A - y_{ij}(IK)] > \\ [A - D_j(00)]^2 = N_{ij}(00) \end{aligned} \tag{11''}$$

Consequently for any ith angle of rotation the following holds true

$$N_{ij}(IK) \geqslant N_{ij}(00) \tag{11'''}$$

The largest quantity of different discrete angles of rotation is determined by the radius vector; since the radius vector at a given scale is the same for (00) and (IK), and equal to $D_j = D_j(00) = D_j(IK)$, the quantity of possible angles of rotation n_i is also the same

$$n_i^2 = 2\pi D_j$$

Consequently, the quantity of coordinate descriptions yielded by all translations at all possible angles of rotation is equal to

$$N_j(00) = \sum_i N_{ij}(00) = \sum_{i=0}^{2\pi D_j} (A - D_j)^2 = 2\pi D_j(A - D_j)^2 \tag{13}$$

$$N_j(IK) = \sum_i N_{ij}(IK) = \sum_{i=0}^{2\pi D_j} [A - x_{ij}(IK)][A - y_{ij}(IK)] \quad (14)$$

But, since the sums (13) and (14) contain an equal quantity of summands for which (11''') holds for every i, it follows that with the ith scale we have

$$N_j(00) = 2\pi D_j(A - D_j)^2 \leqslant$$

$$\sum_{i=0}^{2\pi D_j} [A - x_{ij}(IK)][A - y_{ij}(IK)] = N_j(IK) \quad (14a)$$

or

$$N_j(00) \leqslant N_j(IK) \quad (14')$$

It follows from (9) in particular that the minimum scale is the same for (00) and (IK). Let this minimum scale hold for $j = j_{min}$. The maximum scale, in accordance with (5), is obtained for (00) and $j = j_{max}(00)$ for which

$$D_{j\max}(00) = x_{ij\max}(00) = y_{ij\max}(00) = A - 1 \quad (5')$$

from which, according to (6) $j_{max}(00) = A - 1$.

Thus, the quantity of possible scales for (00) is equal to

$$n_j(00) = j_{max}(00) - j_{min} = (A - 1) - j_{min} \quad (15)$$

The full quantity of coordinate descriptions of the special circle N(00) is obviously equal to the sum of the quantities of coordinate descriptions for each of the possible scales; as may be seen from (15), the number of summands in this sum is equal to $n_j(00) = (A - 1) - j_{min}$.

Consequently

$$N(00) = \sum_{j=j_{min}}^{j_{max}(00)} N_j(00) = \sum_{j=j_{min}}^{A-1} 2\pi D_j(A - D_j)^2 \quad (16)$$

The maximum scale for (IK) is determined in accordance with (5) by the equality

$$x_{ij\max}(IK) = A - 1, \text{ if } y_{ij\max}(IK) \leqslant x_{ij\max}(IK) \quad (5'')$$

or the equality

$$y_{ij\max}(IK) = A - 1, \text{ if } x_{ij\max}(IK) \leqslant y_{ij\max}(IK)$$

But it follows from (11) and (11') that for any angle of rotation for each j we have

$$x_{ij}(IK) \leqslant D_j(IK), \quad y_{ij}(IK) \leqslant D_j(IK)$$

Consequently, these also hold true

$$\begin{aligned} x_{ij\max}(IK) &\leqslant D_{j\max}(IK) \\ y_{ij\max}(IK) &\leqslant D_{j\max}(IK) \end{aligned} \qquad (11'''')$$

Comparing (5') and (5") we have

$$x_{ij\max}(IK) = x_{ij\max}(00) = A - 1 = D_{j\max}(00)$$

when $y_{ij\max}(IK) \leqslant x_{ij\max}(IK)$

$$y_{ij\max}(IK) = y_{ij\max}(00) = A - 1 = D_{j\max}(00) \qquad (17)$$

when $x_{ij\max}(IK) \leqslant y_{ij\max}(IK)$

But, then it follows from (11"") and (17) that $D_{j\max}(IK)$ $D_{j\max}(00)$ or, making use of (6)

$$j_{\max}(IK) \geqslant j_{\max}(00) = A - 1 \qquad (18)$$

The number of possible scales for (IK) is equal, in analogy with (15),

$$n_j(IK) = j_{\max}(IK) - j_{\min} \qquad (19)$$

Comparing (19) and (15) and taking (18) into account, we have

$$n_j(IK) \geqslant n_j(00) \qquad (20)$$

The full quantity of coordinate descriptions of the configuration under study N(IK) is obviously equal to the sum of the quantities of coordinate descriptions of each of the possible scales, that is, taking (19) into consideration

$$\begin{aligned} n(IK) &= \sum_{j=j_{\min}}^{j_{\max}(IK)} N_j(IK) = \\ &\sum_{j=j_{\min}}^{j_{\max}(IK)} \sum_{i=0}^{2\pi D_j} [A - x_{ij}(IK)][A - y_{ij}(IK)] \end{aligned} \qquad (21)$$

Let us represent the sum (21) in the form of two summands

$$\sum_{j=j_{min}}^{j_{max}(IK)} N_j(IK) = \sum_{j=j_{min}}^{j_{max}(00)} N_j(IK) + \sum_{j=j_{max}(00)+1}^{j_{max}(IK)} N_j(IK) = \Sigma_1 + \Sigma_2 \qquad (21')$$

and let us examine the first summand

$$\Sigma_1 = \sum_{j=j_{min}}^{j_{max}(00)} N_j(IK) \qquad (21'')$$

It can be seen from a comparison of (21") and (16) that the numbers of summands is the same in both sums and, in accordance with (14), each term of the sum of (21") is larger than (or equal to) the term with the same summation subscript of (16). Consequently, (22) holds

$$\sum_{j=j_{min}}^{j_{max}(00)} N_j(00) \leqslant \Sigma_1 \qquad (22)$$

Let us examine the second summand of sum (21)

$$\Sigma_2 = \sum_{j=j_{max}(00)+1}^{j_{max}(IK)} N_j(IK) \qquad (21''')$$

The summands of this sum (as in all the sums examined) make sense only when $x_{ij} \leqslant A - 1$ and $y_{ij} \leqslant A - 1$.

It follows from this fact and (18) that

$$\Sigma_2 \geqslant 0 \qquad (23)$$

Consequently, if (22) holds, the same is all the more true of

$$\sum_{j=j_{max}}^{j_{max}(00)} N_j(00) \leqslant \Sigma_1 + \Sigma_2 \qquad (24)$$

In accordance with (16), the left side of this inequality is the full quantity of coordinate descriptions of the special circle N(00); but the right side, according to (21') and (24), is the full

quantity of coordinate descriptions of the configuration under study N(IK). Consequently, according to (24), N(00) $\leq$ N(IK), thus, the left inequality (u) is proved.

The proof of the right inequality of (7) is constructed in a similar manner and does not cause difficulty; in the course of the proof for the number of coordinate descriptions of the special straight line segment with detailedness $\alpha(0\pi) = \alpha(IK)$ we obtain:

$$N(0\pi) = \sum_{j=j_{min}}^{A-1} \sum_{i=0}^{2\pi D_j(0\pi)} \left[A - D_j(0\pi) \cos \frac{1}{D_j(0\pi)}\, i\right] \left[A - D_j(0\pi) \sin \frac{1}{D_j(0\pi)}\, i\right] \qquad (25)$$

where $D_j(0\pi)$ is the length of the segment (0π) on the jth scale.

Submitted 4 December 1959

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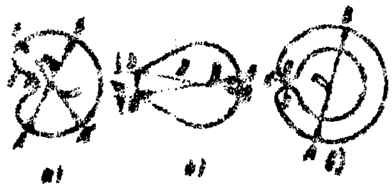
FIGURE APPENDIX



a b c

Figure 1

a

b

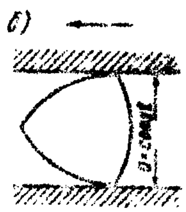


Figure 2

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